

National Association of Special Education Teachers (NASET)

THE PRACTICAL TEACHER

This Month's Topic:

Best Practices in Mathematics

Research-based Top Ten Strategies for Mathematics Achievement

Research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics teaching practices. The following ten instructional practices are from "Improving Student Achievement in Mathematics: Part 1: Research Findings", by Douglas A. Grouws & Kristin J. Cebulla; December 2000 (Updated January 2002).

- 1.) Opportunity to Learn
- 2.) Focus on Meaning
- 3.) Problem Solving
- 4.) Opportunities for Invention and Practice
- 5.) Openness to Student Solutions and Student Interactions
- 6.) Small Group Learning
- 7.) Whole-Class Discussion
- 8.) Focus on Number Sense
- 9.) Use Concrete Materials
- 10.) Use Calculators

1. Opportunity to Learn

Research Says:

The extent of the students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement. Opportunity to learn (OTL) was studied in the First International Mathematics Study (Husén, 1967), where teachers were asked to rate the extent of student exposure to particular mathematical concepts and skills. Strong correlations were found between OTL scores and mean student achievement scores, with high OTL scores associated with high achievement. The link was also found in subsequent international studies, such as the Second International Mathematics Study (McKnight et al., 1987) and the Third International Mathematics and Science study (TIMSS) (Schmidt, McKnight, & Raizen, 1997).

Classroom Implications:

It seems prudent to allocate sufficient time for mathematics instruction at every grade level. Short class periods in mathematics, instituted for whatever practical or philosophical reason, should be seriously questioned. Of special concern are the 30-35 minute class periods for mathematics being implemented in some middle schools.

Textbooks that devote major attention to review and that address little new content each year should be avoided, or their use should be heavily supplemented. Teachers should use textbooks as just one instructional tool among many, rather than feel duty-bound to go through the textbook on a one-section-per-day basis.

It is important to note that opportunity to learn is related to equity issues. Some educational practices differentially affect particular groups of students' opportunity to learn. For example, a recent American Association of University of Women study (1998) showed that boys' and girls' use of technology is markedly different. Girls take fewer computer science and computer design courses than do boys. Furthermore, boys often use computers to program and solve problems, whereas girls tend to use the computer primarily as a word processor. As technology is used in the mathematics classroom, teachers must assign tasks and responsibilities to students in such a way that both boys and girls have active learning experiences with the technological tools employed.

2. Focus on Meaning

Research Says:

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.

There is a long history of research, going back to the work of Brownell (1945,1947), on the effects of teaching for meaning and understanding. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations.

Classroom Implications:

- Emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner.
- Create a classroom learning context in which students can construct meaning.
- Make explicit the connections between mathematics and other subjects.
- Attend to student meanings and student understandings.

3. Problem Solving

Research Says:

Students can learn both concepts and skills by solving problems.

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge.

Classroom Implications:

There is evidence that students can learn new skills and concepts while they are working out solutions to problems. Development of more sophisticated mathematical skills can also be approached by treating their development as a problem for students to solve. Research suggests that it is not necessary for teachers to focus first on skill development and then move on to problem solving. Both can be done together. Skills can be developed on an as-needed basis, or their development can be supplemented through the use of technology. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.

4. Opportunities for Invention and Practice

Research Says:

Giving students both an opportunity to discover and invent new knowledge and an opportunity to practice what they have learned improves student achievement.

Data from the TIMSS video study show that over 90% of mathematics class time in the United States 8th-grade classrooms is spent practicing routine procedures, with the remaining time generally spent applying procedures in new situations. Virtually no time is spent inventing new procedures and analyzing unfamiliar situations. In contrast, students at the same grade level in typical Japanese classrooms spend approximately 40% of instructional time practicing routine procedures, 15% applying procedures in new situations, and 45% inventing new procedures and analyzing new situations.

Research suggests that students need opportunities for both practice and invention. Findings from a number of studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas.

Classroom Implications:

Balance is needed between the time students spend practicing routine procedures and the time they devote to inventing and discovering new ideas. Teachers need not choose between these; indeed, they must not make a choice if students are to develop the mathematical power they need.

To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informal procedures.

5. Openness to Student Solutions and Student Interactions

Research Says:

Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.

Student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics.

Structuring instruction around carefully chosen problems, allowing students to interact when solving problems, and then providing opportunities for them to share their solution methods result in increased

achievement on problem-solving measures. These gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.

Classroom Implications:

Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion.

6. Small Group Learning

Research Says:

Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement.

Davidson (1985) reviewed studies that compared student achievement in small group settings with traditional whole-class instruction. In more than 40% of these studies, students in the classes using small group approaches significantly outscored control students on measures of student performance. In only two of the 79 studies did control-group students perform better than the small group students, and in these studies there were some design irregularities. From a review of 99 studies of cooperative group-learning methods, Slavin (1990) concluded that cooperative methods were effective in improving student achievement. The most effective methods emphasized both group goals and individual accountability.

Classroom Implications:

When using small groups for mathematics instruction, teachers should:

- Choose tasks that deal with important mathematical concepts and ideas.
- Select tasks that are appropriate for group work.
- Consider having students initially work individually on a task and then follow with group work where students share and build on their individual ideas and work.
- Give clear instructions to the groups and set clear expectations for each (for each task or each group?).
- Emphasize both group goals and individual accountability.
- Choose tasks that students find interesting.
- Ensure that there is closure to the group work, where key ideas and methods are brought to the surface either by the teacher or the students, or both.

7. Whole-Class Discussion

Research Says:

Whole-class discussion following individual and group work improves student achievement.

Research suggests that whole class discussion can be effective when it is used for sharing and explaining the variety of solutions by which individual students have solved problems. It allows students to see the many ways of examining a situation and the variety of appropriate and acceptable solutions. Wood (1999) found that whole-class discussion works best when discussion expectations are clearly understood. Students should be expected to evaluate each other's ideas and reasoning in ways that are not critical of the sharer. Students should be expected to be active listeners who participate in discussion and feel a sense of responsibility for each other's understanding.

Classroom Implications:

It is important that whole-class discussion follows student work on problem-solving activities. The discussion should be a summary of individual work in which key ideas are brought to the surface. This can be accomplished through students presenting and discussing their individual solution methods, or through other methods of achieving closure that are led by the teacher, the students, or both.

Whole-class discussion can also be an effective diagnosis tool for determining the depth of student understanding and identifying misconceptions. Teachers can identify areas of difficulty for particular students, as well as ascertain areas of student success or progress.

8. Focus on Number Sense

Research Says:

Teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important.

Number sense relates to having an intuitive feel for number size and combinations, and the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgments. It involves mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the reasonableness of numerical results. Markovits and Sowder (1994) studied 7th-grade classes where special units on number magnitude, mental computation and computational estimation were taught. They determined that after this special instruction, students were more likely to use strategies that reflected sound number sense, and that this was a long-lasting change. In a study of second graders, Cobb (1991) and his colleagues found that students' number sense was improved by a problem-centered curriculum that emphasized student interaction and self-generated solution methods. Almost every student developed a variety of strategies to solve a wide range of problems. Students also demonstrated increased persistence in solving problems.

Classroom Implications:

Competence in the many aspects of number sense is an important mathematical outcome for students. Over 90% of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation or a calculator. However, in many classrooms, efforts to instill number sense are given insufficient attention.

As teachers develop strategies to teach number sense, they should strongly consider moving beyond a unit-skills approach (i.e. a focus on single skills in isolation) to a more integrated approach that encourages the development of number sense in all classroom activities, from the development of computational procedures to mathematical problem-solving.

9. Use Concrete Materials

Research Says:

Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.

In a review of activity-based learning in mathematics in kindergarten through grade 8, Suydam and Higgins (1977) concluded that using manipulative materials produces greater achievement gains than not using them. In a more recent meta-analysis of sixty studies (kindergarten through postsecondary)

that compared the effects of using concrete materials with the effects of more abstract instruction, Sowell (1989) found that the long-term use of concrete materials by teachers knowledgeable in their use improved student achievement and attitudes.

Classroom Implications:

Research suggests that teachers use manipulative materials regularly in order to give students hands-on experience that helps them construct useful meanings for the mathematical ideas they are learning. Use of the same materials to teach multiple ideas over the course of schooling shortens the amount of time it takes to introduce the material and helps students see connections between ideas.

The use of concrete material should not be limited to demonstrations. It is essential that children use materials in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking.

10. Use Calculators

Research Says:

Using calculators in the learning of mathematics can result in increased achievement and improved student attitudes.

Studies have consistently shown that thoughtful use of calculators improves student mathematics achievement and attitudes toward mathematics. From a meta-analysis of 79 non-graphing calculator studies, Hembree and Dessart (1986) concluded that use of hand-held calculators improved student learning. Analysis also showed that students using calculators tended to have better attitudes towards mathematics and better self-concepts in mathematics than their counterparts who did not use calculators. They also found that there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction.

Research on the use of graphing calculators has also shown positive effects on student achievement. Most studies have found positive effects on students' graphing ability, conceptual understanding of graphs and their ability to relate graphical representations to other representations. Most studies of graphing calculators have found no negative effect on basic skills, factual knowledge, or computational skills.

Classroom Implications:

One valuable use for calculators is as a tool for exploration and discovery in problem-solving situations and when introducing new mathematical content. By reducing computation time and providing immediate feedback, calculators help students focus on understanding their work and justifying their methods and results. The graphing calculator is particularly useful in helping to illustrate and develop graphical concepts and in making connections between algebraic and geometric ideas.

In order to accurately reflect their meaningful mathematics performance, students should be allowed to use their calculators in achievement tests. Not to do so is a major disruption in many students' usual way of doing mathematics, and an unrealistic restriction because when they are away from the school setting, they will certainly use a calculator in their daily lives and in the workplace.

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