

National Association of Special Education Teachers (NASET)

THE PRACTICAL TEACHER

This Month's Topic:

School-Wide Strategies for Managing Mathematics

Mathematics instruction is a lengthy, incremental process that spans all grade levels. As children begin formal schooling in kindergarten, they develop ‘number sense’, an intuitive understanding of foundation number concepts and relationships among numbers. A central part of number sense is the student’s ability to internalize the number line as a precursor to performing mental arithmetic. As students progress through elementary school, they must next master common math operations (addition, subtraction, multiplication, and division) and develop fluency in basic arithmetic combinations (‘math facts’). In later grades, students transition to applied, or ‘word’, problems that relate math operations and concepts to real-world situations. Successful completion of applied problems requires that the student understand specialized math vocabulary, identify the relevant math operations needed to solve the problem while ignoring any unnecessary information also appearing in that written problem, translate the word problem from text format into a numeric equation containing digits and math symbols, and then successfully solve.

It is no surprise, then, that there are a number of potential blockers to student success with applied problems, including limited reading decoding and comprehension skills, failure to acquire fluency with arithmetic combinations (math facts), and lack of proficiency with math operations. Deciding what specific math interventions might be appropriate for any student must therefore be a highly individualized process, one that is highly dependent on the student’s developmental level and current math skills, the requirements of the school district’s math curriculum, and the degree to which the student possesses or lacks the necessary auxiliary skills (e.g., math vocabulary, reading comprehension) for success in math. Here are some wide-ranging classroom (Tier I RTI) ideas for math interventions that extend from the primary through secondary grades.

- [Applied Problems: Encourage Students to Draw to Clarify Understanding](#) (Van Essen & Hamaker, 1990; Van Garderen, 2006). Making a drawing of an applied, or ‘word’, problem is one easy heuristic tool that students can use to help them to find the solution. An additional benefit of the drawing strategy is that it can reveal to the teacher any student misunderstandings about how to set up or solve the word problem. To introduce students to the drawing strategy, the

teacher hands out a worksheet containing at least six word problems. The teacher explains to students that making a picture of a word problem sometimes makes that problem clearer and easier to solve. The teacher and students then independently create drawings of each of the problems on the worksheet. Next, the students show their drawings for each problem, explaining each drawing and how it relates to the word problem. The teacher also participates, explaining his or her drawings to the class or group. Then students are directed independently to make drawings as an intermediate problem-solving step when they are faced with challenging word problems. NOTE: This strategy appears to be more effective when used in later, rather than earlier, elementary grades.

- **Applied Problems: Improving Performance Through a 4-Step Problem-Solving Approach** (Pólya, 1957; Williams, 2003). Students can consistently perform better on applied math problems if they follow an efficient 4-step plan of understanding the problem, devising a plan, carrying out the plan, and looking back. (1) UNDERSTAND THE PROBLEM. To fully grasp the problem, the student may restate the problem in his or her own words, note key information, and identify missing information. (2) DEVISE A PLAN. In mapping out a strategy to solve the problem, the student may make a table, draw a diagram, or translate the verbal problem into an equation. (3) CARRY OUT THE PLAN. The student implements the steps in the plan, showing work and checking work for each step. (4) LOOK BACK. The student checks the results. If the answer is written as an equation, the student puts the results in words and checks whether the answer addresses the question posed in the original word problem.
- **Math Computation: Boost Fluency Through Explicit Time-Drills** (Rhymer, Skinner, Jackson, McNeill, Smith & Jackson, 2002; Skinner, Pappas & Davis, 2005; Woodward, 2006). Explicit time-drills are a method to boost students' rate of responding on math-fact worksheets. The teacher hands out the worksheet. Students are told that they will have 3 minutes to work on problems on the sheet. The teacher starts the stop watch and tells the students to start work. At the end of the first minute in the 3-minute span, the teacher 'calls time', stops the stopwatch, and tells the students to underline the last number written and to put their pencils in the air. Then students are told to resume work and the teacher restarts the stopwatch. This process is repeated at the end of minutes 2 and 3. At the conclusion of the 3 minutes, the teacher collects the student worksheets. TIPS: Explicit time-drills work best on 'simple' math facts requiring few computation steps. They are less effective on more complex math facts. Also, a less intrusive and more flexible version of this intervention is to use time-prompts while students are working independently on math facts to speed their rate of responding. For example, at the end of every minute of seatwork, the teacher can call the time and have students draw a line under the item that they are working on when that minute expires.
- **Math Computation: Motivate With 'Errorless Learning' Worksheets** (Caron, 2007). Reluctant students can be motivated to practice math number problems to build computational fluency when given worksheets that include an answer key (number problems with correct answers) displayed at the top of the page. In this version of an 'errorless learning' approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. Such speed drills build computational fluency while promoting students' ability to visualize and to use a mental number line. TIP: Consider turning this activity into a 'speed drill'. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

- **Math Computation: Two Ideas to Jump-Start Active Academic Responding** (Skinner, Pappas & Davis, 2005). Research shows that when teachers use specific techniques to motivate their classes to engage in higher rates of active and accurate academic responding, student learning rates are likely to go up. Here are two ideas to accomplish increased academic responding on math tasks. First, break longer assignments into shorter assignments with performance feedback given after each shorter ‘chunk’ (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional ‘natural’ reinforcer. Second, allow students to respond to easier practice items orally rather than in written form to speed up the rate of correct responses.
- **Math Homework: Motivate Students Through Reinforcers, Interesting Assignments, Homework Planners, and Self-Monitoring** (Bryan & Sullivan-Burstein, 1998). Improve students’ rate of homework completion and quality by using reinforcers, motivating ‘real-life’ assignments, a homework planner, and student self-monitoring. (1) Reinforcers: Allow students to earn a small reward (e.g., additional free time) when they turn in all homework assignments for the week. (2) ‘Real-life’ Assignments: Make homework meaningful by linking concepts being taught to students’ lives. In a math lesson on estimating area, for example, give students the homework task of calculating the area of their bedroom and estimating the amount of paint needed to cover the walls. (3) Homework Planner: Teach students to use a homework planner to write down assignments, organize any materials (e.g., worksheets) needed for homework, transport completed homework safely back to school, and provide space for parents and teachers to communicate about homework via written school-home notes. (4) Student Self-Monitoring: Direct students to chart their homework completion each week. Have students plot the number of assignments turned in on-time in green, assignments not turned in at all in red, and assignments turned in late in yellow.
- **Math Instruction: Consolidate Student Learning During Lecture Through the Peer-Guided Pause** (Hawkins, & Brady, 1994). During large-group math lectures, teachers can help students to retain more instructional content by incorporating brief Peer Guided Pause sessions into lectures. Students are trained to work in pairs. At one or more appropriate review points in a lecture period, the instructor directs students to pair up to work together for 4 minutes. During each Peer Guided Pause, students are given a worksheet that contains one or more correctly completed word or number problems illustrating the math concept(s) covered in the lecture. The sheet also contains several additional, similar problems that pairs of students work cooperatively to complete, along with an answer key. Student pairs are reminded to (a) monitor their understanding of the lesson concepts; (b) review the correctly math model problem; (c) work cooperatively on the additional problems, and (d) check their answers. The teacher can direct student pairs to write their names on the practice sheets and collect them as a convenient way to monitor student understanding.
- **Math Instruction: Increase Student Engagement and Improve Group Behaviors With Response Cards** (Armendariz & Umbreit, 1999; Lambert, Cartledge, Heward & Lo, 2006). Response cards can increase student active engagement in group math activities while reducing disruptive behavior. In the group-response technique, all students in the classroom are supplied with an erasable tablet (‘response card’), such as a chalk slate or laminated white board with erasable marker. The teacher instructs at a brisk pace. The instructor first poses a question to the class. Students are given sufficient wait time for each to write a response on his or her response card. The teacher then directs students to present their cards. If most or all of the class has the correct answer, the teacher praises the group. If more than one quarter of the students records an incorrect answer on their cards, however, the teacher uses guided questions and demonstration to steer students to the correct answer.

- **[Math Instruction: Maintain a Supportive Atmosphere for Classroom “Math Talk”](#)** (Cooke & Adams, 1998). Teachers can promote greater student ‘risk-taking’ in mathematics learning when they cultivate a positive classroom atmosphere for math discussions while preventing peers from putting each other down. The teacher models behavioral expectations for open, interactive discussions, praises students for their class participation and creative attempts at problem-solving, and regularly points out that incorrect answers and misunderstandings should be celebrated—as they often lead to breakthroughs in learning. The teacher uses open-ended comments (e.g., “What led you to that answer?”) as tools to draw out students and encourage them to explore and apply math concepts in group discussion. Students are also encouraged in a supportive manner to evaluate each other’s reasoning. However, the teacher intervenes immediately to prevent negative student comments or ‘put-downs’ about peers. As with any problem classroom behavior, a first offense requires that the student meet privately with the instructor to discuss teacher expectations for positive classroom behavior. If the student continues to put down peers, the teacher imposes appropriate disciplinary consequences.
- **[Math Instruction: Support Students Through a Wrap-Around Instruction Plan](#)** (Montague, 1997; Montague, Warger & Morgan, 2000). When teachers instruct students in more complex math cognitive strategies, they must support struggling learners with a ‘wrap-around’ instructional plan. That plan incorporates several elements: (a) Assessment of the student’s problem-solving skills. The instructor first verifies that the student has the necessary academic competencies to learn higher-level math content, including reading and writing skills, knowledge of basic math operations, and grasp of required math vocabulary. (b) Explicit instruction. The teacher presents new math content in structured, highly organized lessons. The instructor also uses teaching tools such as Guided Practice (moving students from known material to new concepts through a thoughtful series of teacher questions) and ‘overlearning’ (teaching and practicing a skill with the class to the point at which students develop automatic recall and control of it). (c) Process modeling. The teacher adopts a ‘think aloud’ approach, or process modeling, to verbally reveal his or her cognitive process to the class while using a cognitive strategy to solve a math problem. In turn, students are encouraged to think aloud when applying the same strategy—first as part of a whole-class or cooperative learning group, then independently. The teacher observes students during process modeling to verify that they are correctly applying the cognitive strategy. (d) Performance feedback. Students get regular performance feedback about their level of mastery in learning the cognitive strategy. That feedback can take many forms, including curriculum-based measurement, timely corrective feedback, specific praise and encouragement, grades, and brief teacher conferences. (e) Review of mastered skills or material. Once the student has mastered a cognitive strategy, the teacher structures future class lessons or independent work to give the student periodic opportunities to use and maintain the strategy. The teacher also provides occasional brief ‘booster sessions’, reteaching steps of the cognitive strategy to improve student retention.
- **[Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling](#)** (Baxter, Woodward & Olson, 2005). Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class ‘math journals’. Journaling is a valuable channel of communication about math issues for students who are unsure of their skills and reluctant to contribute orally in class. At the start of the year, the teacher introduces the journaling assignment, telling students that they will be asked to write and submit responses at least weekly to teacher-posed questions. At first, the teacher presents ‘safe’ questions that tap into the students’ opinions and attitudes about mathematics (e.g., ‘How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?’). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students’ own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical

symbols, and diagrams in their journal entries to enhance their explanations. The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class on the general quality and content of class journal responses. Regular math journaling can prod students to move beyond simple 'rote' mastery of the steps for completing various math problems toward a deeper grasp of the math concepts that underlie and explain a particular problem-solving approach. Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

- **Math Problem-Solving: Help Students Avoid Errors With the 'Individualized Self-Correction Checklist'** (Zrebiec Uberti, Mastropieri & Scruggs, 2004). Students can improve their accuracy on particular types of word and number problems by using an 'individualized self-instruction checklist' that reminds them to pay attention to their own specific error patterns. To create such a checklist, the teacher meets with the student. Together they analyze common error patterns that the student tends to commit on a particular problem type (e.g., 'On addition problems that require carrying, I don't always remember to carry the number from the previously added column.'). For each type of error identified, the student and teacher together describe the appropriate step to take to prevent the error from occurring (e.g., 'When adding each column, make sure to carry numbers when needed.'). These self-check items are compiled into a single checklist. Students are then encouraged to use their individualized self-instruction checklist whenever they work independently on their number or word problems. As older students become proficient in creating and using these individualized error checklists, they can begin to analyze their own math errors and to make their checklists independently whenever they encounter new problem types.
- **Math Review: Balance Massed & Distributed Practice** (Carnine, 1997). Teachers can best promote students acquisition and fluency in a newly taught math skill by transitioning from massed to distributed practice. When students have just acquired a math skill but are not yet fluent in its use, they need lots of opportunities to try out the skill under teacher supervision—a technique sometimes referred to as 'massed practice'. Once students have developed facility and independence with that new math skill, it is essential that they then be required periodically to use the skill in order to embed and retain it—a strategy also known as 'distributed practice'. Teachers can program distributed practice of a math skill such as reducing fractions to least common denominators into instruction either by (a) regularly requiring the student to complete short assignments in which they practice that skill in isolation (e.g., completing drill sheets with fractions to be reduced), or (b) teaching a more advanced algorithm or problem-solving approach that incorporates—and therefore requires repeated use of—the previously learned math skill (e.g., requiring students to reduce fractions to least-common denominators as a necessary first step to adding the fractions together and converting the resulting improper fraction to a mixed number).
- **Math Review: Teach Effective Test-Preparation Strategies** (Hong, Sas, & Sas, 2006). A comparison of the methods that high and low-achieving math students typically use to prepare for tests suggests that struggling math students need to be taught (1) specific test-review strategies and (2) time-management and self-advocacy skills. Among review-related strategies, deficient test-takers benefit from explicit instruction in how to take adequate in-class notes; to adopt a systematic method to review material for tests (e.g., looking over their notes each night, rereading relevant portions of the math text, reviewing handouts from the teacher, etc.), and to give themselves additional practice in solving problems (e.g., by attempting all homework items, tackling additional problems from the text book, and solving problems included in teacher handouts). Deficient test-takers also require pointers in how to allocate and manage their study

time wisely, to structure their study environment to increase concentration and reduce distractions, as well as to develop 'self-advocacy' skills such as seeking additional help from teachers when needed. Teachers can efficiently teach effective test-preparation methods as a several-session whole-group instructional module.

- **Math Vocabulary: Preteach, Model, and Use Standard Math Terms** (Chard, D., n.d.). Three strategies can help students to learn essential math vocabulary: preteaching key vocabulary items, modeling those vocabulary words, and using only universally accepted math terms in instruction. (1) Preteach key math vocabulary. Math vocabulary provides students with the language tools to grasp abstract mathematical concepts and to explain their own reasoning. Therefore, do not wait to teach that vocabulary only at 'point of use'. Instead, preview relevant math vocabulary as a regular part of the 'background' information that students receive in preparation to learn new math concepts or operations. (2) Model the relevant vocabulary when new concepts are taught. Strengthen students' grasp of new vocabulary by reviewing a number of math problems with the class, each time consistently and explicitly modeling the use of appropriate vocabulary to describe the concepts being taught. Then have students engage in cooperative learning or individual practice activities in which they too must successfully use the new vocabulary—while the teacher provides targeted support to students as needed. (3) Ensure that students learn standard, widely accepted labels for common math terms and operations and that they use them consistently to describe their math problem-solving efforts.
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